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ON THE PASSAGE OF LOW FREQUENCY ELECTROMAGNETIC WAVES

THROUGH THE IONOSPHERIC PLASMA

by

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SUMMARY

A rigorous solution is obtained of the problem of low frequency electro-
magnetic wave passage through a plane-stratified magnetoactive ionospheric
plasma in case of longitudinal propagation (the angle α between the wave vector
$\vec{k}$ and the direction of the external magnetic field being zero).

The transmission and reflection factors are found on the basis of the
above solution in the frequency band 1.5 – 100 kc/sec for the daytime and night-
time ionosphere models. It is shown that in the case under consideration the
geometrical optics approximation does not coincide with the rigorous solution
of the wave equation in frequencies < 10 kc/sec, whereupon this discrepancy
increases with frequency decrease.

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INTRODUCTION

A great interest has been manifest during the last few years in connection
with the investigation of whistlers and of various forms of VLF radioemission
arriving on the Earth from the upper regions of the atmosphere relative to the
propagation of low frequency electromagnetic waves in the ionosphere [1]. This
interest is not casual, for the study of the indicated events may provide new
valuable information on physical properties of the ionosphere (concentration,
collision of electrons, influence of the geomagnetic field on the propagation
of long and ultralong radiowaves and others) and, in particular of its upper
regions.

One of the important problems arising during the theoretical consideration
of the problem of VLF electromagnetic wave propagation in the ionosphere consists
in the quantitative estimate of the value of the transmission factor for waves
of the indicated band passing through a magnetoactive ionospheric plasma.

Attempts to find a rigorous solution of this problem, when the angle α between the wave vector $\vec{k}$ and the direction of the Earth's magnetic field is arbitrary and may vary in the general case, are beset with significant mathematical difficulties (even if the ionosphere sphericity is disregarded) and so far have failed to lead to a positive result.

The geometrical optics approximation has been applied in the series of works [2, 3] for the calculation of the value of the transmission factor for VLF electromagnetic waves passing through the ionosphere. However, such an approach can hardly be considered sufficiently substantiated, for in the long-wave part of the VLF-band the criterion of applicability of the geometrical optics approximation $(c/\omega)(|d\tilde{n}/dz|/|\tilde{n}|^2) \ll 1$ $(\tilde{n} = n - j\varkappa$ is the complex index of refraction) is knowingly not fulfilled in the lower regions of the ionosphere. Thus, a simple numerical calculation shows that in frequencies of several kc/s the quantity $(c/\omega)(|d\tilde{n}/dz|/|\tilde{n}|^2) \simeq 1$ at altitudes from 60 to 100 km for the daytime ionosphere and from 85 to 150 km for the nighttime ionosphere.

In the present work a rigorous solution of the problem is obtained for VLF electromagnetic waves' passage through a magnetoactive ionospheric plasma in the case of longitudinal propagation (α = 0). The values of the transmission and reflection factors in the frequency range from 1.5 to 100 kc/sec are found on the basis of that solution for the daytime and night models of the ionosphere.

1. STATEMENT OF THE PROBLEM

Let us consider a model of plane-stratified ionospheric plasma, of which the parameters are dependent on only one coordinate $\underline{z}$, and let us assume that the external magnetic field H_0 is directed perpendicularly to the layers (along the axis $\underline{z}$). Thus, the wave equation [4]

$$\frac{d^2 F_{1,2}}{dz^2} + \frac{\omega^2}{c^2}[n(z) - j\varkappa(z)]_{1,2}^2 F_{1,2} = 0, \tag{1}$$

is valid for the case of longitudinal propagation of plane electromagnetic waves in the plasma (α = 0); here

$$F_{1,2} = E_x \pm jE_y;$$

$$(n - j\varkappa)_{1,2}^2 = 1 - \frac{v}{1 - js \mp \sqrt{u}} \tag{2}$$

is the square of the complex index of refraction for extraordinary (index 1; upper sign ahead of the radical) and ordinary waves (index 2; lower sign ahead of the radical). In formula (2), which was written without taking into account the motions of molecules and ions, parameters $\underline{v}$, $\underline{u}$ and $\underline{s}$ are determined by the well known relations: $v = \omega_0^2/\omega^2$, $u = \omega_H^2/\omega^2$, $s = v_{\ni\phi}/\omega$, where $\omega_0 = \sqrt{4\pi e^2 N/m}$ is the plasma frequency; $\omega_H = eH_0/m_c$ is the electron gyrofrequency; $N = N(z)$ is the concentration of electrons; $v_{\ni\phi} = v_{\ni\phi}(z)$ is the effective collision frequency of electrons with other particles; e and m are respectively the charge and the mass of the electron.

Analysis of formula (2) shows that the extraordinary wave will be the object of substantially weaker damping by comparison with the ordinary wave which, as will be shown below, does not practically seep through the ionosphere.

Eq.(1) may be integrated by numerical methods for given dependences of concentration $N(z)$ and collision frequency $v_{э\phi}(z)$ of electrons on altitude for the day and night ionosphere models.

2. BOUNDARY CONDITIONS. CALCULATED FORMULAS

Let us so choose the ionosphere boundary $z = z_0$ that for $z < z_0$ the complex index of refraction $\underline{n}$ convert to the unity (free space conditions). We shall consider the plasma layer $z_0 < z < z_1$, the upper boundary of which is situated at a certain altitude $z = z_1$ in the ionosphere, and assume that a plane electromagnetic wave is incident upon the ionosphere from below (along the axis z). It follows from boundary conditions for tangential vector components of electric and magnetic foelds that function F and its derivative dF/dz must be continuous during the transitions through the levels $z = z_0$ and $z = z_1$. For $z < z_0$ and a wave incident upon the ionosphere from below we may write (time dependence taken in the form $e^{j\omega t}$).

$$F^{inc}(z) = Ae^{-j(\omega/c)(z-z_0)},\qquad(3)$$

and for a wave reflected from the lower ionosphere boundary,

$$F^{ref}(z) = Be^{j(\omega/c)(z-z_0)}.\qquad(4)$$

Utilizing (3) and (4) with boundary conditions for $z = z_0$, we shall obtain

$$A + B = F(z_0),$$
$$-j\frac{\omega}{c}(A - B) = F'(z_0),\qquad(5)$$

where $F(z_0)$ is the solution of Eq.(1) at $z = z_0$; $F' = dF/dz$. From (5) we shall find the expression for the complex index of refraction R of electromagnetic waves from the lower boundary of the ionosphere:

$$R = \frac{B}{A} = \frac{F(z_0) - j(c/\omega)F'(z_0)}{F(z_0) + j(c/\omega)F'(z_0)}.\qquad(6)$$

The question is more complex with boundary conditions in the plane $z = z_1$, for, generally speaking, the aggregate field in this plane will be determined not only by the wave having traversed the plasma layer $z_0 \leqslant z \leqslant z_1$, but also by the waves reflected from higher regions of the ionosphere ($z > z_1$). However, this difficulty may be overcome as follows. Assume that the level $z = z_1$ is situated at a sufficient height in the ionosphere so that for $z \geqslant z_1$ the conditions of applicability of geometric optics approximation are fulfilled*.

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* Our computations in the $1.5 - 100$ kc/sec range have shown that the geometric optics approximation is valid in the entire altitude region $z \geqslant 100$ km for the daytime model and $z \geqslant 150$ km for the night ionosphere model.

Then for $z \geqslant z_1$ we may limit ourselves to the consideration of an upward propagating wave, having traversed the layer, and which has in the indicated approximation the form (for more details see [5])

$$F^{pas}(z) = \frac{M}{\sqrt{\tilde{n}(z)}} e^{-j(\omega/c)\int_{z_1}^{z} \tilde{n}(z)dz}, \tag{7}$$

where M is an arbitrary constant.

Utilizing (7) and the boundary conditions in the plane $z = z_1$, we obtain

$$M / \sqrt{\tilde{n}(z_1)} = F(z_1),$$

$$-j\frac{\omega}{c}\frac{M\tilde{n}(z_1)}{\sqrt{\tilde{n}(z_1)}}\left[1 - j\frac{c}{2\omega}\frac{d\tilde{n}/dz}{\tilde{n}^2(z)}\right]_{z=z_1} = F'(z_1). \tag{8}$$

When the applicability condition of geometric optics approximation is fulfilled, we may neglect the second addend in the bracket by comparison with the unity. Denoting $M / \sqrt{\tilde{n}(z_1)} = M_1$, we shall finally obtain

$$\begin{aligned} F(z_1) &= M_1, \\ F'(z_1) &\simeq -j(\omega/c)M_1\tilde{n}(z_1), \end{aligned} \right\} \tag{9}$$

where M_1 is an arbitrary constant.

Therefore, having assigned ourselves at $z = z_1$ the amplitude M_1 of the wave having traversed the layer, we shall be in a position to integrate Eq. (1) in the interval $z_0 \leqslant z \leqslant z_1$ and to compute by formula (6) the value of the reflection factor R. The ratio d of amplitude of passed wave to that of the wave incident upon the ionosphere may, as is easy to show, be determined by the expression

$$d = \frac{M_1}{A} = \frac{2M_1}{F(z_0) + j(c/\omega)F'(z_0)}. \tag{10}$$

Since the differential equation (1) is linear, while the quantities R and d depend only on the ratios $F'(z_0) / F(z_0)$ and $F(z_1) / F(z_0)$, and not on the function F itself or on its derivative, the value of M_1 in (9) may be chosen absolutely arbitrarily. Usually the value of M_1 is chosen starting from the considerations of practicability of numerical integration of Eq. (1) by computer.

The transmission factor of electromagnetic waves passing through the ionosphere will be defined as the ratio of the time-average value of energy flux for the passed wave to the corresponding value of energy flux for the incident wave:

$$D = \overline{S^{pass}} / \overline{S^{inc}}, \tag{11}$$

where, as is well known [6],

$$\bar{\vec{S}} = (c/8\pi)\,\mathrm{Re}\,[\vec{E}\vec{H}^*] \tag{12}$$

(here and subsequently the star will be used for denoting complex-conjugate quantities).

Making use of the second Maxwellian equation for complex-conjugate vectors $\vec{E}^*$ and $\vec{H}^*$

$$\mathrm{rot}\,\vec{E}^* = j(\omega/c)\vec{H}^*, \tag{13}$$

we substitute $\vec{H}^*$ in (12) in correspondence with (13). If we subsequently introduce instead of electric field components the function F (see (1)), it is not difficult to show that in the case of longitudinal propagation considered the expression

$$\bar{S} = \frac{c}{8\pi}\,\mathrm{Re}\left(-j\frac{c}{\omega}F\frac{dF^*}{dz}\right). \tag{14}$$

is valid for the value of the averaged energy flux. In correspondence with (14) and with the help of (3) and (7), we shall find

$$\bar{S}^{\mathrm{inc}} = (c/8\pi)\,|A|^2, \tag{15}$$

$$\bar{S}^{\mathrm{pas}}(z) = \frac{c}{8\pi}\frac{|M|^2}{|\tilde{n}(z)|}e^{-(2\omega/c)\int_{z_1}^{z}\varkappa(z)dz}\,\mathrm{Re}\left\{\tilde{n}^*(z)\left(1+j\frac{c}{2\omega}\frac{d\tilde{n}^*/dz}{[\tilde{n}^*(z)]^2}\right)\right\}. \tag{16}$$

Neglecting for $z \gtrsim z_1$ the second addend in parentheses of formula (16) (see for comparison (8)), we have

$$\bar{S}^{\mathrm{pass}}(z) = \frac{c}{8\pi}\frac{|M|^2\,n(z)}{|\tilde{n}(z)|}e^{-(2\omega/c)\int_{z_1}^{z}\varkappa(z)dz} \tag{17}$$

From (11), (15) and (17) we finally obtain the expression for the transmission factor D:

$$D(z) = \frac{4\,|M|^2\,n(z)}{|F(z_0)+j(c/\omega)F'(z_0)|^2\,|\tilde{n}(z)|}e^{-(2\omega/c)\int_{z_1}^{z}\varkappa(z)dz} \tag{18}$$

3. RESULTS OF CALCULATIONS

Eq.(1) was integrated by the Runge-Kutta method with the aid of a computer in the altitude range from 50 to 100 km for the daytime model and from 80 to 200 km for the night ionosphere model. The dependences of concentration and

of the effective number of electron collisions on altitude, utilized during computations (see Fig.1), are rather close to those known in literature [7, 2, 3]. The value H_0 of the magnetic field was given in the dipole approximation for the geomagnetic latitude of 50°. The integration of Eq.(1) and the calculations of the reflection and transmission factors by formulas (6), (18) were carried out in the frequency range 1.5 − 100 kc/sec. The boundary
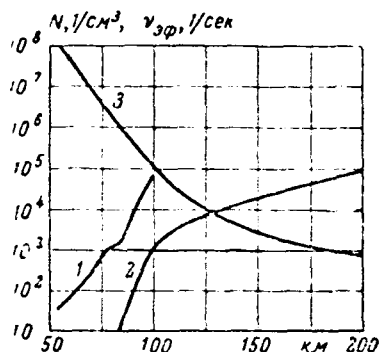

Fig.1. Dependence of electron concentration and of the effective number of electron collisions on (3) on altitude (1 - for the daytime ionosphere model and 2 - for the nighttime.
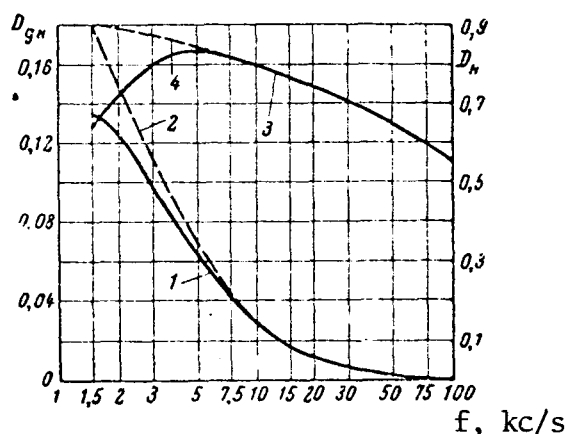
Fig.2. Dependence of the transmission factor on frequency for the daytime ($D_{дн}$, 1) and nighttime $D_{н}$, 3) ionosphere models. The dashed curves 2 and 4 correspond to the results of calculations in the geometric optics approximation.

conditions (9) were given respectively at 100 km altitude for the daytime model and 200 km for the nighttime model, whereupon it was preliminarily established by way of calculation that the conditions of applicability of geometric optics approximation are fulfilled at greater heights over the entire frequency band considered. The values of the transmission factors were computed for the same altitudes. The results of calculations are plotted in Figs.2, 3 & 4. The dependences of the transmission factor D on frequency for the extraordinary wave are brought out in Fig.2. As may be seen from it, the value of the transmission factor for the daytime ionosphere model decreases with the rise of frequency, beginning from the value $D \simeq 0.13$ in the frequency $f = 1.5$ kc/sec, whereupon in the frequency $f = 100$ kc/sec the transmission factor is practically zero. The value of D for the night model ionosphere is significantly greater than for the daytime and attains its maximum value in the frequency $f \simeq 4$ kc/sec*. Note that according to the reciprocity theorem [4], the curves for the dependence of transmission factor on frequency brought out in Fig.2 remain in force if the wave is incident upon the ionosphere from above and the amplitude of the incident wave is respectively given at 100 and 200 km altitude for the day and night ionosphere models.

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* For $f > 100$ kc/sec the value of the transmission factor through the night ionosphere decreases monotonically with the rise of frequency and vanishes at the frequency of 1 Mc/sec.

A comparison of the results of calculations of the values of the transmission factor obtained by way of integration of Eq.(1) using the method considered by us and in the geometric optics approximation is of great interest. It is not difficult to see (refer to (18)) that in such an approximation the transmission factor is determined as

$$D^{geom}(z) = \frac{n(z)}{|\tilde{n}(z)|} e^{-(2\omega/c)\int_{z_\bullet}^{z} \varkappa(z)dz}.$$
(19)

The results of calculations of D^{geom} are plotted by dashes in Fig.2; it follows from them that in the frequency $f < 10$ kc/sec the geometric optics approximation gives overrated values of the transmission factor, whereupon the departure of D^{geom} from the rigorous solution of Eq.(1) increases with the decrease of frequency. It should also be noted that in case of night model ionosphere the approximate method does not make apparent the extremum in the vicinity of the frequency $f \simeq 4$ kc/sec.
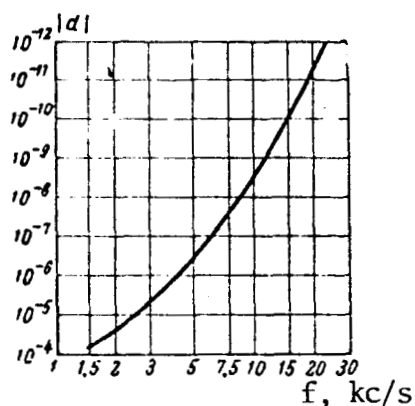

Fig.3. Dependence of the modulus of amplitude ratio of passed wave to the amplitude of the wave incident upon the ionosphere on frequency (daytime model, ord. wave).
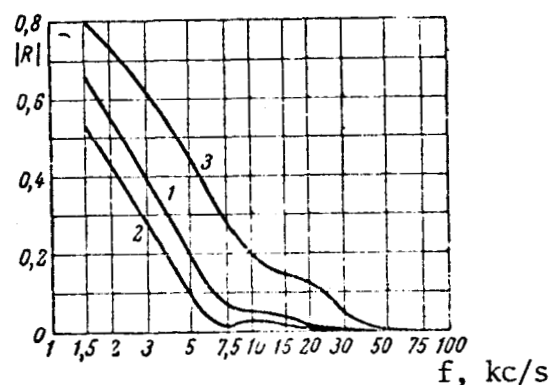
Fig.4. Dependence of the modulus of the reflection factor on frequency for the daytime (1 - extraordinary wave, 3 - ordinary wave) and the night (2 - extraordinary wave) ionosphere models.

The ordinary wave does not practically leak through the ionosphere and the notion of passed energy flux (and consequently of the transmission factor) loses any meaning for it. Plotted in Fig.3 are the results of calculations of the modulus of the quantity d (see (10)) for the daytime model ionosphere, which show that the electric field of the ordinary wave having reached 100 km height, weakens by no less than 10^4 times by comparison with the corresponding value of the field of the ionosphere-incident wave, as early as in the frequency $f = 1.5$ kc/sec. As the frequency increases, this weakening of the field increases rapidly. Analogous results of calculations were obtained for the case of passage of an ordinary wave through the night ionosphere (they are not shown in Fig.3).

The results of calculations of the modulus of the reflection factor $|R|$ are represented in Fig.4, from which it may be seen that the quantity $|R|$ decreases with the rise of frequency. As was to be expected, the least reflection takes place for the extraordinary wave in the case of the night model ionosphere. Attention is drawn by the similitude of the curves in Fig.4. This is apparently the consequence of the fact that the reflected field is basically formed in the initial region of the ionosphere, where the difference in the shape of the curves $n(z)$ and $\kappa(z)$, respectively for the daytime and night ionosphere models, is immaterial.

The results of calculations of $|R|$ for the case of reflection of the ordinary wave from the night model ionosphere are not presented, since they are analogous to the results of calculations for the daytime model.

C O N C L U S I O N S

1. As a result of rigorous solution of the wave equation for the case of longitudinal propagation ($\alpha = 0$) of VLF electromagnetic waves in the ionospheric plasma it is shown that in nighttime 55 to 80 percent of incident energy flux may pass through the ionosphere to altitudes of the order of 200 km in the frequency range from 1.5 to 100 kc/sec. The transmission factor through the daytime ionosphere attains 13 percent in the frequency of 1.5 kc/sec (to 100 km altitude); as the frequency increases, its value decreases monotonically to 1% in the frequency of 20 kc/sec. The electromagnetic waves of 100 kc frequency do not practically pass through the daytime ionosphere.

For the estimate of the complementary damping of VLF electromagnetic waves at altitudes above 100 km for the daytime and 200 km for the night ionosphere the geometric optics approximation may be utilized (see (7)).

2. Theoretical calculations have shown the presence of a maximum in the vicinity of $f \simeq 4$ kc/sec on the curve of transmission factor's dependence on frequency for the night model ionosphere. This may possibly be one of the causes conditioning a lower decrement of spectral component damping of whistlers lying in the frequency band from 3 to 5 kc/sec by comparison with the remaining frequencies. The phenomenon of such a weak damping in the indicated frequency range was first revealed by Storey [8].

3. The solution obtained in geometric optics approximation does not coincide with the rigorous solution of the wave equation in frequencies below 10 kc/sec, whereupon this discrepancy increases as the frequency decreases. In the frequency of 1.5 kc/sec the value of the transmission factor found by the geometric optics method is 1.3 to 1.4 times higher than the corresponding value obtained by way of rigorous solution of the wave equation. This case should be borne in mind when considering the results of the works [2, 3], in which the geometric optics approximation was utilized for the calculation of the value of the transmission factor for VLF electromagnetic waves' passage through the ionosphere.

4. The proposed method of calculation of the transmission factor for VLF electromagnetic waves' passage through the ionosphere may be extended to the case of lower frequencies (less than 1.5 kc/sec), when the motion of ions and electrons has to be taken into account.

In conclusion the author considers it to be his duty to express his gratitude to L. A. Zhekulin for his interest in the work and valuable comments, and also to V. D. Gus'kova, who composed the program and completed the calculations on the BESM-2 computer.

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